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IMPLICATIONS OF A HEAVY TOP IN SUPERSYMMETRIC THEORIES

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ABSTRACT

In the context of the radiative electroweak symmetry breaking scenario, we investigate the implications of a heavy top quark mass, close to its infrared fixed point, on the low energy parameters of the minimal supersymmetric standard model. We use analytic expressions to calculate the Higgs masses as well as the supersymmetric masses of the third generation. We further assume bottom-tau unification at the GUT scale and examine the constraints put by this condition on the parameter space ($\tan\beta$,α_3), using the renormalization group procedure at the two-loop level. We find only a small fraction of the parameter space where the above conditions can be satisfied, namely $1 \le \tan\beta \le 2$, while $0.111 \le \alpha_3(M_Z) \le 0.118$. We further analyse the case where all three Yukawa couplings reach the perturbative limit just after the unification scale. In this latter case, the situation turns out to be very strict demanding $\tan\beta \sim 63$.

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Introduction

The last years there has been a revived interest in the supergravity unified models and their low energy effective theories, mainly due to the fact that LEP measurements are in good agreement with a gauge coupling constant unification scenario with supersymmetric β-function coefficients down to the scale of $\sim$ 1TeV. However, the existence of supersymmetry will only be confirmed when new particles – the superpartners of the standard model spectrum – will be observed in (near) future experiments. Thus, the study of supersymmetric grand unification is very important and should be seen in conjuction with the predictions for the new particles which may be observed soon. So far, the constraints put by the unification of the three gauge couplings require a superpartner mass spectrum in the range of $(0.1-1)$TeV which can be accessible in the near future.

There is another experimental fact the last few years which seems to be related with the fate of the electroweak symmetry breaking in an effective supersymmetric low energy theory. The non-observation of the top quark gives a lower bound on its mass $m_t \gtrsim 100$GeV. Although this result is disappointing from the experimental point of view, on the other hand, it fits perfectly with the idea of radiative symmetry breaking scenario suggested several years ago [1, 2]. Indeed the renormalization group impoved SUSY Higgs potential breaks the $[SU(2)\times U(1)]_{EW}$ symmetry when the top Yukawa coupling is large enough to drive one of the soft supersymmetry breaking parameters (namely $m^2_{H_2}$) negative.

Grand unification based on the most popular groups, with the minimal number of fermion and Higgs content, implies additional relations in the initial values of the parameters of the theory. Thus, for example in the Yukawa sector, one such well known constraint requires the bottom and tau lepton Yukawa couplings h_b and h_τ, to be equal at the unification scale E_G

$$h_b(E_G) = h_\tau(E_G) \tag{1}$$

In certain cases, and particularly in string derived unified models, additional constraints on the Yukawa sector of the theory are often obtained, i.e.

$$h_b(E_G) \approx h_\tau(E_G) \approx h_t(E_G) \sim g_{String} \tag{2}$$

where h_t is the top Yukawa coupling and g_{String} is the value of the unified gauge coupling at the string scale $E_{String} \geq E_G \sim 10^{16}$GeV. In patricular, a large top Yukawa coupling which is implied by the last equality in Eq.(2), motivates again the study of the fixed point solutions proposed several years ago in the context of non supersymmetric theories [3].

Motivated by the experimental fact that the top quark mass is rather high as well as from the aforementioned theoretical speculations, in the present

work we wish to study the implications of the above considerations on the low energy theory. In order to minimize the arbitrary parameters and to avoid complications with flavour changing neutral currents, we assume universality of the scalar mass parameters at the GUT scale. Using renormalization group techniques, we derive the mass formulae of the scalar masses (in particular those affected by a large top quark mass) and examine their properties close to the infrared fixed point of the top mass. Furthermore we investigate the regions of the parameter $\tan\beta =< H_2 > / < H_1 >$ which are compatible with the above constraints and the minimization conditions put by the renormalization group improved Higgs potential.

Radiative Symmetry Breaking in the Presence of a Heavy Top Quark

One of the most appealing features of supergravity theories is the radiative symmetry breaking mechanism [2] which may occur in the presence of a heavy top quark mass. Indeed, the renormalization group improved Higgs potential breaks the electroweak symmetry if the top Yukawa coupling is large enough to drive the $m^2_{H_2}$ mass parameter negative below a certain scale Q_0.

At the tree level the supersymmetric Higgs potential can be written as follows

$$\begin{aligned}\mathcal{V}_0(Q) &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_3^2(\epsilon_{ij}{H_1}^i{H_2}^j + h.c.)\\ &\quad +\frac{1}{8}(g^2+g'^2)\left(|H_1|^2-|H_2|^2\right)^2 + \frac{1}{2}g^2|H_1^{i*}H_2^i|^2 \ , \end{aligned} \tag{3}$$

where $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ are the standard Higgs superfields and ϵ_{ij} is the antisymmetric tensor in two dimensions. We have also introduced the two Higgs mass parameters

$$m_1^2 = m_{H_1}^2 + \mu^2, \tag{4}$$
$$m_2^2 = m_{H_2}^2 + \mu^2. \tag{5}$$

Finally $m_{H_{1,2}}$ and m_3 are the soft SUSY breaking mass terms and μ is the Higgs mixing mass parameter.

The above tree-level potential $\mathcal{V}_0(Q)$ depends strongly on the energy scale Q. It has been shown [4] however, that a correct minimization procedure can be achieved (making the potential relatively stable), if one includes the one-loop corrections $\Delta\mathcal{V}_1(Q)$

$$\Delta\mathcal{V}_1(Q) = \frac{1}{64\pi^2}\mathrm{Str}\left[\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{Q^2}-\frac{3}{2}\right)\right], \tag{6}$$

where $\mathcal{M}^2$ is the field dependent tree level mass matrix squared. Thus finally

$$\mathcal{V}_H(Q) = \mathcal{V}_0(Q) + \Delta\mathcal{V}_1(Q) \tag{7}$$

The symbol *Str* stands for the supertrace which is defined as follows

$$\mathrm{Str} f(\mathcal{M}^2) = \sum_i q_i (-1)^{2s_i} (2s_i + 1) f(m_i^2) \tag{8}$$

with q_i being the color degrees of freedom while m_i and s_i are the mass and the spin of the corresponding particle.

Now, electroweak symmetry breaking occurs if the following two conditions are met:

- The supersymmetric Higgs potential should develop an asymmetric minimum below some scale $Q \leq Q_0$. This requirement is expressed by the condition
$$m_1^2(Q) m_2^2(Q) - m_3^4(Q) \leq 0. \tag{9}$$
- The Higgs potential should be bounded from below. This requirement sets the second condition, which reads
$$m_1^2(Q) + m_2^2(Q) \geq 2|m_3(Q)|^2 \tag{10}$$

The minimization conditions $\frac{\partial \mathcal{V}_H}{\partial v_i} = 0$, where $v_i \equiv < H_i >$, result the well known equations

$$\frac{1}{2} M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1} \tag{11}$$

$$\frac{1}{2}\sin 2\beta = -\frac{m_3^2}{\mu_1^2 + \mu_2^2} \tag{12}$$

where we have introduced the new mass parameters $\mu_i^2 = m_{H_i}^2 + \mu^2 + \sigma_i^2$, which take into account the corrections to the Higgs potential from the one-loop contributions σ_i^2

$$\sigma_i^2 \equiv \frac{\partial \Delta \mathcal{V}_1}{\partial v_i} \tag{13}$$

From the above equations one can conclude that the one-loop corrections to the Higgs potential appear in the minimization conditions through shifts of the Higgs mass parameters $m_i^2 \to m_i^2 + \sigma_i^2$. It has been shown [5] , that although 31 particles contribute to σ_i^2 corrections, there are finally large cancellations which reduce significantly their effect to the electroweak symmetry breaking. Moreover, the one-loop contribution of the t-squark-quark sector to the masses of the neutralinos, Higgsinos and gauginos seems to be well below the 10% [6] (except in the unfavorable case of a very light tree-level mass)

The most important contributions arize from the squarks of the third generation and the top quark mass. Therefore, it is obvious that the Higgs mass parameters m_{H_i} and the t-squarks play an important role in the minimization of the Higgs potential.

The scale dependence of m_{H_i} and t-squarks is given by the renormalization group equations which can be integrated to give the following results. The m_{H_1} Higgs mass parameter is given by

$$m_{H_1}^2 = m_0^2 + C_{H_1}(t) m_{1/2}^2 \tag{14}$$

where $t = \ln Q$, m_0 and $m_{1/2}$ are the universal scalar and gaugino mass parameters at E_G, and $C_{H_1} \sim 0.57$ for $t \sim \ln M_Z$. For the rest of the scalar masses, denoting for convenience $m_{\tilde{t}_L} \equiv \tilde{m}_1$, $m_{\tilde{t}_R} \equiv \tilde{m}_2$ and $m_{H_2} \equiv \tilde{m}_3$, we can write the general analytic form [7]

$$\tilde{m}_n^2 = \alpha_n m_0^2 + C_n(t) m_{1/2}^2 - n\delta_m^2(t) - n\delta_A^2(t) \tag{15}$$

where α_n depends on the Käller manifold and hereafter we assume that α_n=1. The quantities $\delta_{m,A}^2(t)$ are given by

$$\delta_m^2(t) = \left(\frac{m_t(t)}{2\pi v \gamma_Q(t) \sin\beta}\right)^2 \times (3m_0^2 I(t) + m_{1/2}^2 J(t)) \tag{16}$$

and,

$$\delta_A^2(t) \;\; = \;\; \Delta_A^2(t) - \frac{3}{2}\left(\frac{m_t(t)}{2\pi v \gamma_Q(t) \sin\beta}\right)^2 E_A^2(t) \tag{17}$$

where $v = 246$GeV and I, J, Δ_A^2 and E_A^2, are integrals containing functions of gauge couplings, i.e.

$$I \;\; = \;\; \int_t^{t_0} dt' \gamma_Q^2(t') \tag{18}$$

$$J \;\; = \;\; \int_t^{t_0} dt' \gamma_Q^2(t') C(t') \tag{19}$$

$$\Delta_A^2 \;\; = \;\; \int_t^{t_0} \frac{h_t^2(t')}{8\pi^2} A^2(t')\, dt' \tag{20}$$

$$E_A^2 \;\; = \;\; \int_t^{t_0} dt' \gamma_Q^2(t') \Delta_A^2(t') \tag{21}$$

with $t_0 = \ln E_G$, $C(t) \equiv \sum_{n=1}^3 C_n(t)$, while $\gamma_Q(t) = \prod_{j=1}^3 (\alpha_{j,0}/\alpha_j)^{c_Q^j/2b_j}$. Clearly, a large top mass implies also a large value of the top-Yukawa coupling, and therefore the negative contributions δ^2 will become also significant. It is possible then to have $\tilde{m}_3^2 \equiv m_{H_2}^2$ negative, and the radiative symmetry breaking scenario will take place.

A very interesting possibily arizes in the case where the top mass is close to its infrared quasi-fixed point. The evolution of the top quark coupling, assuming all the way down supersymmetry, is given by

$$h_t = \frac{h_t(t_0)\gamma_Q(t)}{(1+\frac{3}{4\pi^2}h_t^2(t_0)I(t))^{1/2}} \tag{22}$$

In this case, i.e. for $\frac{h_t^2(t_o)}{4\pi} \sim 1$, since $I(t \sim lnm_t) \gg 1$, we can approximate the above

$$h_t(fixed) \approx \frac{2\pi}{\sqrt{3I(t)}}\gamma_Q(t) \tag{23}$$

i.e. independent of the initial value $h_t(t_0)$. Thus $m_t(fixed) = m_t^0 \sin\beta \sim (190-200) sin\beta$ GeV, dependening on the precise values of α_3, E_G etc.

The scalar masses of $m_{\tilde{t}_L}$, $m_{\tilde{t}_R}$ and the Higgs which couples to the up-quarks, take a very simple m_t-independent form in this case. For $h_t = h_t(fixed)$ Eq.(15) simplifies to

$$\tilde{m}_n^2 = (1-\frac{n}{2})m_0^2 + \Big[C_n(t) - \frac{n}{6}\frac{J}{I}\Big]m_{1/2}^2 \tag{24}$$

As far as one assumes $A(E_G) \leq 3|m_0|$, corrections due to A-contributions to the above formula have been found to be very small and thus they have been totally ignored. Calculation of the various t-dependent quantities at $t \simeq \ln m_t$ gives [8, 7]

$$C_1(t) \simeq 5.30, \quad C_2(t) \simeq 4.90, \quad C_3 \simeq .57, \quad I \simeq 113, \quad J \simeq 590 \tag{25}$$

There are some worth-noting properties of the above mass formulae. Indeed, first note that $m_{\tilde{t}_R}$ depends only on $m_{1/2}^2$ up to A corrections which have been found nigligible. A second property is that the dependence of the sum $m_{\tilde{t}_L}^2 + m_{H_3}^2$ on m_0^2, vanishes at the limit $m_t \to m_t(fixed)$. The above properties have also been noticed in Ref[9]. It is interesting to see the implications of the above simplified formulae in the case of the minimization conditions of the Higgs potential. We start first with the Higgs mixing parameter μ involved in the minimization conditions Eqs.(11, 12). Ignoring one-loop effects for simplicity, the μ parameter can be given in terms of the known parameters I, J, the unkown Higgs-vev ratio and the initial values $(m_0, m_{1/2})$, by the following equation

$$|\mu| = \frac{1}{\sqrt{2}}\Big\{\frac{k^2+2}{k^2-1}m_0^2 + \Big(\frac{k^2}{k^2-1}\frac{J}{I} - 1\Big)m_{1/2}^2 - M_Z^2\Big\}^{1/2} \tag{26}$$

with $k = \tan\beta$. In Fig.(1),we plot the $|\mu|$-values in the parameter space $(m_0, m_{1/2})$, for $\tan\beta = 1.1$ and $\tan\beta = 5$. For the most of the parameter

space, $|\mu| \leq 1.5$TeV. Of course, as $\tan\beta \to 1$, μ grows larger, and a fine tuning problem may arize in Eq.(11), in order to obtain the correct experimetal value of M_Z. Thus, to avoid fine tuning, we may put the condition on $\tan\beta \geq 1.1$, which finally translates to the bound $M_t \equiv m_t(pole) \geq (150-155)$GeV.

The μ-parameter plays also important role in the squark mass matrices. In particular, the t-squark mass matrix is

$$M_Q^2 \quad = \quad \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^2 & M_{RR}^2 \end{pmatrix} \tag{27}$$

with eigenvalues given by

$$m^2_{\tilde{t}_{1,2}} = \frac{M_{LL}^2 + M_{RR}^2 \pm \sqrt{\left(M_{LL}^2 - M_{RR}^2\right)^2 + 4M_{LR}^4}}{2} \tag{28}$$

where

$$\begin{aligned} M_{LL}^2 + M_{RR}^2 &= \frac{1}{2}m_0^2 + (C_1 + C_2 - \frac{J}{2I})m_{1/2}^2 + 2m_t^2 + \frac{1}{2}m_Z^2 cos2\beta \\ M_{LL}^2 - M_{RR}^2 &= \frac{1}{2}m_0^2 + (C_1 - C_2 + \frac{J}{6I})m_{1/2}^2 + (\frac{4}{3}M_W^2 - \frac{5}{6}M_Z^2)cos2\beta \\ M_{LR}^2 &= m_t^0\,(A\sin\beta + \mu\cos\beta) \end{aligned}$$

In Fig.(2) (assuming $\mu > 0$), we plot contours of the above eigenmasses in the parameter space $(m_0, m_{1/2})$ for the choice $A(E_G) = -\sqrt{3}m_0$, and two representative values of $\tan\beta$ in the low range $(1.1-10)$, namely $\tan\beta = 1.1$ and $\tan\beta = 5$. In most of the parameter space the light eigenstate preserves the independence of m_0 mass parameter. For reasonable initial values of the parameters m_0 and $m_{1/2}$, the squark masses are well bellow the 1TeV, and therefore accessible to future experimets.

Notice finally that the one-loop contributions to the effective potential will also result to a shift in the $|\mu|$ parameter. Making use of the fact that in the limit $m_{1/2} \gg m_0$ we can approximate

$$\ln\frac{m^2_{\tilde{t}_1}}{M_Z^2} \sim \ln\frac{m^2_{\tilde{t}_2}}{M_Z^2} \sim \ln\frac{< m^2_{\tilde{t}} >}{M_Z^2}$$

we may obtain an analytic form for the one-loop corrected $|\mu|$ parameter, when m_t and $m_{\tilde{t}_{1,2}}$ corrections are taken into account [5]

$$|\mu| = \sqrt{(\mu_0^2 + \eta^2)/(1-\Omega^2)} \tag{29}$$

where

$$\eta^2 \quad = \quad \frac{\alpha_2}{8\pi\cos^2\theta_W}\left\{\left[\left(M_{LL}^2 + M_{RR}^2\right)\left(\frac{1}{4} - \rho^2\right)\right.\right.$$

$$
\begin{aligned}
&+ \quad \left(M_{LL}^2 - M_{RR}^2\right)\left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right) - \rho^2 A^2\Big]\left(\ln\tilde{\rho}^2 - 1\right) \\
&- \quad 2m_t^2\left(\ln\rho^2 - 1\right)\frac{\rho^2}{k^2}\Big\}\frac{k^2+1}{k^2-1} \qquad (30)
\end{aligned}
$$

$$
\Omega^2 = \frac{\alpha_2}{8\pi\cos^2\theta_W}\left\{\frac{\rho^2(k^2+1)}{k^2(k^2-1)}\right\}\left(\ln\tilde{\rho}^2 - 1\right) \qquad (31)
$$

with $\rho = m_t/M_Z$, $\tilde{\rho} =< m_{\tilde{t}} > /M_Z$ and μ_0 the tree level parameter defind in (26). For moderate values of $m_{1/2}$ however, these corrections are not going to alter substantially our previous results.

Bottom–Tau Yukawa Unification and the IR Fixed Point

One of the great successes of the most popular GUTs is the equality of the bottom and tau Yukawa couplings at the GUT scale which lead to the correct prediction of the experimentally determined relation $m_b \approx 3\, m_\tau$ at low energies. Several groups [10] have examined the effects of h_b, h_τ relations implied by various unified theories, assuming minimal supersymmetry with grand unification at an energy scale close to 10^{16}GeV. It has been claimed that the GUT relation $h_b = h_\tau$ implies a heavy top quark with a value of the Yukawa coupling close to its infrared fixed point. In this section, we wish to present a detailed numerical analysis in the context of the GUT constraints mentioned above. We will mainely discuss the constraints on the parameter space $(\tan\beta, \alpha_3)$ when bottom-tau Yukawa unification is assumed and examine the connection of this constraint in relation with the top-mass. We will further examine the case where the three Yukawa couplings reach the perturbative limit just after the unification scale. Our analysis will be done at the two-loop level, taking into account the contribution of the Yukawa couplings, and in particular that of the h_t into the running of the gauge coupling constants. Our results largely agree with previous analyses, however the allowed region in the parameter space $(\tan\beta, \alpha_3)$ is more constrained. In particular, we find that $h_b = h_\tau$ can be satisfied only in a small region of $1 \leq \tan\beta \leq 2$ and $.111 \leq \alpha_3 \leq .118$. The case where all three Yukawa couplings are equal at E_G occurs theoretically in (minimal) $SO(10)$ [11] and in $SU(4) \times SU(2)_L \times SU(2)_R$ [12] . The allowed region of $\tan\beta$ shortens around the value 63 for that case while $\alpha_3(M_Z)$ stays on the lower edge of the experimentally allowed region ($\sim .11$).

We shall present now a detailed description of the procedure we are following. We adopt the so called bottom-up approach starting from M_Z. Which are the inputs at this energy level?

- the experimentally known values of α, $sin^2\theta_W$ and α_3, or equivalently of the three gauge couplings α_i, $i = 1, 2, 3$. The relatively small exper-

imental errors on α and $sin^2\theta_W$ permit us to talk about the "bands" of α_1 and α_2 in the running of those couplings while we treat α_3 as a "free" parameter, inside its experimental limits of course.

- the value of $\tan\beta$, starting its rôle when we reach the energy E_S where SUSY is valid.
- the value of the h_t Yukawa coupling of the top quark. Essentially it is a free parameter as long as it gives the mass of the top quark in the allowed experimental region (110–190)GeV. We use the 1 loop QCD corrections to define the pole mass M_t of the top quark

$$M_t = \frac{h_t(M_t)v/\sqrt{2}}{1+\frac{4}{3\pi}\alpha_3(M_t)}$$

- the values of h_b and h_τ, taken from the relations

$$m_b(m_b) = \frac{h_b(M_Z)v/\sqrt{2}}{\eta_b}, \qquad m_\tau = h_\tau(M_Z)v/\sqrt{2}$$

  We take the mass of the bottom quark $m_b(m_b) = (4.15-4.35)$GeV while that of the τ lepton $m_\tau(m_\tau) = 1.7841$GeV. The factor η_b, appearing in the mass of the bottom quark, includes the 1 loop QCD corrections from m_b to M_Z.

Between M_Z and E_S, which we take to be 1TeV, we run the couplings with the β-functions of the S.M. At E_S we apply the following boundary conditions for the Yukawa couplings

$$h_t^S = \frac{h_t}{\sin\beta}, \quad h_b^S = \frac{h_b}{\cos\beta} \quad \text{and} \quad h_\tau^S = \frac{h_\tau}{\cos\beta}$$

Then onwards we run the couplings using the MSSM β-functions. At an energy E_G, around 10^{16}GeV, the "bands" of the couplings α_1 and α_2 meet and determine what we call "the unification band". The strong coupling $\alpha_3(M_Z)$ should be chosen so that it passes through this unification band in order to achieve gauge coupling unification. A short comment is in order at this point. Since we are using 2 loop β-functions the differential equations for all the couplings are coupled (this fact shows its presence even harder when we demand one or all the Yukawa couplings to grow large at E_G). Therefore the unification band is not uniquely determined but depends, though not strongly, on the particular choice for α_3 as well as on the Yukawa couplings at M_Z. We try to find the values of $\tan\beta$ that permit the growth of h_t to the perturbative limit ($h_t \sim 3.5$) at the energy scale E_G or later, checking always that α_3 passes through the unification band. At the same time we try

to unify, again at E_G, the other two Yukawa couplings: $h_b(E_G) = h_\tau(E_G)$. This could be achieved by varying the mass of the bottom quark inside its experimentally allowed region. Finally we try to arrange the possibility that all thre Yukawa couplings grow to the perturbative limit at E_G. This last step could be achieved by using large values of $\tan\beta$.

We approach, step by step, the above three points, constraining in each step the allowed region of the parameter space of our inputs. In Fig.3 we plot, for several values of the mass of the top quark M_t, $\tan\beta$ versus $\alpha_3(M_Z)$ demanding gauge coupling unification and $h_t(E_G) \lesssim 3.5$. Let us explain the features of the graph. The lower limit on $\alpha_3(M_Z)$ appears because the lower the gauge couplings the larger the slope dh_t/dt (recall that gauge and Yukawa couplings have opposite contributions to the β-functions). This fact permits h_t to grow very fast and reach the perturbative limit before gauge coupling unification is achieved. The same line of thought explains the slope of the "lines" in Fig.3. Choosing a higher $\alpha_3(M_Z)$ we need a higher initial point $h_t^S(E_S)$ to reach the perturbative limit, therefore we need a smaller $\tan\beta$. The turning edges of each line is more intriguing. At the right end the value of $\alpha_3(M_Z)$ is so high that, although h_t permits gauge coupling unification, $\alpha_3(E_G)$ passes above the unification band of (α_1, α_2). Choosing a higher value of $\tan\beta$ (therefore smaller h_t) $d\alpha_i/dt$ grows to larger values. The coupling α_2 receives the biggest contribution, the unification band shifts to higher values and allows α_3 to pass through it. Of course, in that case $h_t(E_G) < 3.5$. For each line in Fig.3, the region where $h_t(E_G) < 3.5$ (dashed lines) grows bigger as m_t grows. Similar arguments explain the left end of the lines. Now $\alpha_3(M_Z)$ is so small that very easily drops below the unification band. Choosing a somewhat higher $\alpha_3(M_Z)$ permits a higher $\tan\beta$. Again, in this case, $h_t(E_G) < 3.5$. Therefore the allowed region for each M_t is inside the envelope–like shape.

Our next step is to demand $h_b(E_G) = h_\tau(E_G)$. For each M_t, we plot in Fig.3 a band (shaded region) corresponding to $m_b(m_b) = 4.15\text{GeV}$ (lower line of the band) and to $m_b(m_b) = 4.35\text{GeV}$ (upper line of the band). We notice that b-τ unification requires M_t to be near its fixed point, being closer for $M_t \sim (150 - 160)\text{GeV}$.

The last step is to require all three Yukawa couplings to reach the perturbative limit near E_G. To achieve that point we need a large value for $\tan\beta$ (for h_b and h_τ) and a large M_t (for h_t) The situation is very strict. For example, using as inputs

$$\tan\beta = 63.4, \quad \alpha_3(M_Z) = .112, \quad M_t = 190GeV, \quad and \quad m_b(m_b) = 4.2GeV$$

we get gauge coupling unification at $E_G = (10^{16.0} - 10^{16.1})\text{GeV}$, while the three Yukawa couplings at the scale $10^{16.2}\text{GeV}$ reach values in the range

$3.1 - 3.5$. Trying to achieve those large values of h_t, h_b and h_τ with a lower value of M_t, one needs to choose either a lower value of $\alpha_3(M_Z)$ or a smaller $\tan\beta$. The latter does not help since, at such large values, $\sin\beta$ does not change much while $\cos\beta$ does, preventing h_b and h_τ to reach the perturbative limit. On the other hand, the change of $\alpha_3(M_Z)$ does not affect h_τ in contrast with h_t and h_b. The situation is greatly complicated since all Yukawa couplings are large.

In conclusion, in this paper we examined the implications of a heavy top quark, and bottom tau unification at the GUT scale, implied by popular unified models, in the minimal supersymmetric standard model. We have assumed a top Yukawa coupling close to its infrared fixed point and we have given analytic forms of the t-squark masses and the Higgs mass parameter responsible for the radiative electroweak symmetry breaking scenario. We have found that $m_{\tilde{t}_L}$ does not depend on the m_0 mass parameter, while all masses under consideration are very weakly dependent on the trilinear scalar parameter A. The bottom-tau unification turned out to be very restrictive. We have found only small ranges in the $(\tan\beta, \alpha_3)$-plane where this condition can be satisfied. Moreover, this condition demands a heavy top with a mass close to its infrared fixed point, as was previously assumed. In the case of the large $\tan\beta$ scenario, the above requirements can be satisfied only in a tiny region with $\tan\beta \approx 63$.

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Figure Captions

Fig.1. Surfaces of constant $|\mu|$ in the parameter space $(m_0, m_{1/2})$. The upper surface corresponds to $\tan\beta = 1.1$ while the lower one corresponds to $\tan\beta = 5$.

Fig.2. Contours of constant $m_{\tilde{t}_{1,2}}$ in the parameter space $(m_0, m_{1/2})$, for two values of $\tan\beta = 1.1$ and 5. (a) and (c) corresponds to the lighter eigenstate while (b) and (d) to the heavier one.

Fig.3. Allowed regions in the space of $(\tan\beta, \alpha_3(M_Z))$, in order to achieve gauge coupling unification, for several values of the top mass M_t. Below the solid part of the contours, h_t reaches the perturbative limit before gauge coupling unification, while below the dashed part α_3 passes above the unification band of α_1 and α_2. Demanding $h_b = h_\tau$ at E_G, the allowed regions, for each M_t value, shrink to the corresponding shaded bands.